POWER SPECTRUM OF TANGENTIAL STRESS ON A WALL IN A TURBULENT FLOW

A. V. Chinak

UDC 532.517.4

The mechanism of formation of the tangential stress spectrum on a wall in a developed turbulent flow is considered. An interpretation of spectra obtained experimentally is proposed. It is shown that the spectrum contains information on the viscous sublayer thickness and enables us to perform nonprecision measurements of tangential stress on the wall in the developed turbulent flow with no precalibration of sensors.

In practice it is often necessary to obtain information on a flow with a method that is sufficiently simple in realization and introduces no disturbances into the flow. One possibility for this is measuring tangential friction on a wall using an electrodiffusion method [1]. Measurements [2–4] show that the spectrum of tangential stress on the wall and the spectrum of velocity pulsations in the viscous wall region differ from the velocity spectrum in the core of the turbulent flow. In [2] the spectrum of wall friction is compared with the results of a linear analysis. This comparison shows the adequacy of the model for describing the high-frequency portion of the spectrum. To model pulsations in the wall region, use is made of experimental data on the pressure spectrum [5] and the wave structure of velocity pulsations [6]. The spectra are represented in dimensionless variables: $n/\overline{S}_0$; $W\overline{S}_0/(\overline{s}_x)^2$(where n is the frequency, $(\overline{s}_x)^2$ is the root-mean-square pulsations of tangential stress on the wall in the direction of the flow, $\overline{S}_0 = V^{*2}/\nu$, V^* is the dynamic velocity), which enable us to generalize data obtained at different Re.

In [7] results of investigations of turbulence in the wall region in a tube with Re = 8700 are given. Profiles of the average velocity and spectra of velocity pulsations in a viscous sublayer and outside it are presented. Data on pulsation and average characteristics obtained in gas-liquid bubble flows are given in [3, 4]. In a dimensionless representation the spectra of tangential stress on a wall in a lowering bubble flow hardly differ from single-phase spectra. In [8], to describe the field of velocity pulsations between the wall and the region of a developed turbulent flow, use is made of a simplified linearized form of the equations of motion; Klebanoff's experimental data [9] are used as boundary conditions for the pulsation component of velocity on the external boundary of the viscous sublayer.

The aim of the present work is to elucidate, by using simple considerations, the dependence of the form of the tangential stress pulsation spectrum on the wall on the averaged characteristics of the flow.

We consider the propagation of velocity disturbances from the external boundary of the viscous layer toward the wall. For simplicity, we consider the viscous sublayer to be a thin layer of liquid at the wall of thickness δ, with this layer having negligibly small inertia effects, and the average velocity profile is linear in y (Fig. 1). Experiments show that with an increase in the dimensionless frequency $n/\overline{S}_0$ from 10^{-3} to 10^{-1} the spectral density of pulsations of tangential stress on the wall decreases approximately 1000-fold. In its physical meaning the dimensionless frequency in this case is the ratio of the thickness of the viscous sublayer to the wavelength of the disturbance. Taking into account the above-said, we will deal with disturbances with a wavelength much greater than the thickness of the viscous sublayer and consider the flow as uniform along the direction of flow. Then in the boundary-layer approximation for the two-dimensional case we can write the equation of motion in the simplified linearized form

$$\frac{\partial V}{\partial t} = -\frac{I}{\rho}\frac{\partial P}{\partial x} + \nu\frac{\partial^2 V}{\partial y^2} \tag{1}$$

Institute of Thermal Physics, Novosibirsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 67, Nos. 5-6, pp. 387-391, November-December, 1994, Original article submitted May 14, 1992; revision submitted February 1, 1994.

 1062-0125/94/6756-1026$12.50

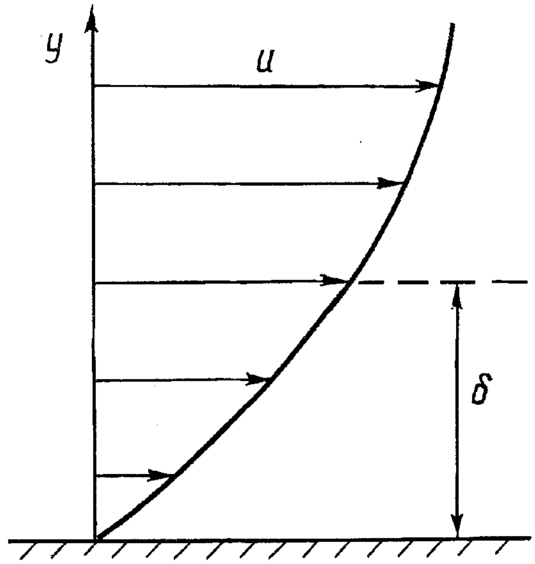

Fig. 1. Scheme of flow in the viscous wall region.

with the boundary conditions

$$V = 0 \quad \text{at} \quad y = 0\,, \qquad V = \overline{V}_\delta + V' \quad \text{at} \quad y = \delta\,. \tag{2}$$

The linearity of the equation of motion enables us to deal separately with the Fourier components of the pulsation velocity V'. This system of equations is satisfied by the following solution:

$$\overline{V} = (V_\delta/\delta)\, y + V_1\, (\exp\,(ky) \cos\,(nt + ky) - \cos\,(nt))\,, \tag{3}$$

$$\frac{\partial P}{\partial x} = -\, V_1 \rho n \sin\,(nt)\,,$$

where $k = \sqrt{n/2\nu}$; V_1 is a factor that makes the solution consistent with the boundary condition on the external boundary and has the form

$$V_1 = V_0/(\exp\,(k\delta) \cos\,(nt_1 + k\delta) - \cos\,(nt_1))\,,$$

where $t_1 = (1/n) \arctan\,(\sin\,(k\delta)/(\exp\,(-k\delta) - \cos\,(k\delta))$; V_0 is the velocity pulsation amplitude with the frequency n on the external boundary of the layer. Then for pulsations of the tangential stress on the wall we have

$$s_x = -\,\mu \frac{\partial\,(V'_{y=0})}{\partial y} = -\,\mu V_1 k\, (\cos\,(nt) - \sin\,(nt))\,.$$

By squaring and time-averaging this expression, for the power spectrum we find

$$W = (\mu V_1 k)^2\,. \tag{4}$$

The expression obtained is a transmission function that describes the relationship between velocity pulsations in the viscous sublayer and tangential stress pulsations on the wall, depending on the frequency, sublayer thickness, and properties of the liquid. Two free parameters remain that refer to the conditions on the external boundary: V_0 is the amplitude of the velocity pulsation harmonic with the frequency n on the external boundary; δ is the distance from the wall to the external boundary of the layer in question. In principle, we are entitled to specify the boundary conditions at any distance from the wall provided that the above assumptions of a small

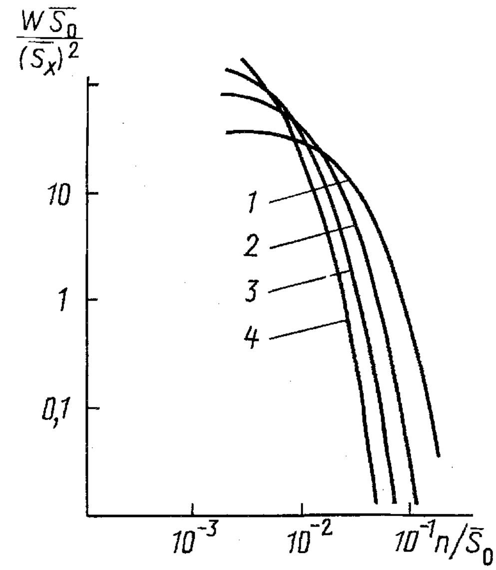

Fig. 2. Calculated power spectra of tangential stress on the wall: 1) $\delta^+ = 8$; 2) 12; 3) 16; 4) 20.

deviation of the average velocity profile from the linear one and small inertia effects are satisfied. Now we define δ more precisely as the distance at which cascade energy transfer to small scales practically ceases and the velocity pulsation spectrum forms mainly by dissipation of the harmonics separately. This region will be referred to as the effective viscous sublayer, whose thickness is comparable to the viscous sublayer thickness. The meaning of this definition for δ is that in this region we can introduce some simple assumptions concerning the velocity pulsation spectrum and need not resort to experiment in specifying boundary conditions on the external boundary.

Velocity spectra obtained outside the viscous sublayer for $y^+ = 10$, 20, 40 and given in [7] have a flat low-frequency region, and the drop in the transmission function obtained by us begins at frequencies that are rather low compared to dissipative vortices and has a fairly steep character. Therefore we can assume that it is the transmission function (4) that is of crucial importance in the formation of velocity spectra in the viscous sublayer and tangential stress spectra on the wall, and the shape of the velocity spectrum on the external boundary does not introduce a large correction and can be considered as flat, i.e.,

$$V_0 = \text{const}.$$

The value of this constant is eliminated from consideration in normalizing the spectrum:

$$W' = W \Big/ \left(\int_0^\infty W \, dn \right).$$

An indirect corroboration of different mechanisms of velocity spectrum formation in the viscous sublayer and outside it is the fact that the appropriate dimensionless variables that generalize experimental data are found only for the spectrum in the viscous sublayer [7]. The assumptions made enable us to establish the relationship between the shape of the spectrum of tangential stress pulsations on the wall and some effective thickness δ of the viscous sublayer. Figure 2 gives diagrams of the spectra, constructed for different δ, in dimensionless coordinates.

We performed measurements in channels of different geometry: a plane channel (2 mm thick), a widening channel between parallel flat plates (the distance between the plates is 2 mm, the apex angle is 12°, and the distance between inclined walls at the measurement point is 25-90 mm) and a narrowing channel of the same geometry [10]. The measurements were performed by an electrodiffusion method [1]. We performed experiments using different sensors that confirmed the possibility of using a sensor with dimensions of 20 μm in the direction of the flow and 100 μm in the direction perpendicular to the flow. Results of measurements for different flows and Reynolds numbers in a dimensionless representation differ little.

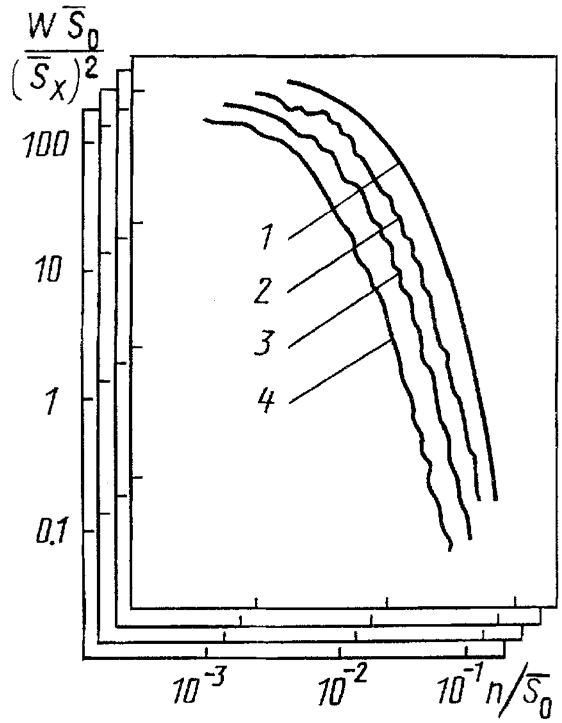

Fig. 3. Comparison of the power spectra of tangential stress on the wall in a plane channel with the calculation: 1) calculation, $\delta^+ = 14.5$; 2) Re = 400; 3) Re = 5500; 4) Re = 9560.

A comparison with the experiments shows that for a certain value of the parameter δ we observe good agreement of the model with experiment. For single-phase turbulent flows in the plane channel (Fig. 3) or the tube this value was equal to $\delta^+ \simeq 14$. Agreement of the model with the experiments is observed even with Re = 2700, constructed from the average velocity and channel thickness. Somewhat different data were obtained in the narrowing channels, where $\delta^+ \simeq 16$. For developed turbulent flow in the widening channel we found that $\delta^+ = 14$.

Data for the two-phase flow [3, 4] are also readily generalized and correspond to the calculated curve for $\delta^+ = 14.5$. Measurements were performed in a vertical tube 42 mm in diameter with velocities $V = 0.5-1.25$ m/sec. The gas content varied within $\beta = 2-15\%$. We performed measurements in a lowering flow by an electrodiffusion method.

A comparison of the above model with data obtained in a liquid with a polymer added [2] yields $\delta^+ \approx 10$. But for this system we observe substantial differences in the character of the spectra, which manifest themselves mainly in the high-frequency region. This result comes as no surprise and demonstrates the inapplicability of the above model to non-Newtonian liquids.

The data obtained enable us to conclude that in most cases the above assumptions concerning spectrum formation in the wall region are quite reasonable; the effective thickness of the viscous sublayer δ obtained by considering the spectra is proportional to the viscous sublayer thickness and the value of tangential stress on the wall and is equal to $\delta^+ \simeq 14.5$. This enables us to perform nonprecision measurements of tangential stress on the wall over the pulsation spectrum. These measurements can be of importance for practical applications in flows where the calibration of sensors is hindered or requirements on accuracy are low. The accuracy of these measurements depends on the algorithm for comparing the measurements with calculations, the frequency characteristic of the measuring system, and the accuracy of determination of δ requires further investigation. It is pertinent to note that the above reasoning refers mainly to the high-frequency portion of the spectrum since at low frequencies the transmission of velocity pulsations to the wall occurs with almost no damping and, consequently, this spectral portion carries information on velocity pulsations in the flow's core.

The author expresses his thanks to the Soros Foundation, which has financed the work.

NOTATION

n, frequency, sec^{-1}; S_x, x-component of tangential stress on the wall, Pa; s_x, x-component of pulsations of tangential stress on the wall, Pa; V, velocity, m/sec; V', velocity pulsations, m/sec; $V^* = (|\bar{S}_x|/\rho)^{1/2}$, dynamic velocity; ν, kinematic viscosity, m^2/sec; μ, dynamic viscosity, kg/(m·sec); P, pressure, Pa; t, time, sec; ρ, density, kg/m^3; W, spectral density, (N/m^2)^2; β, volume gas content, %; $y^+ = V^*y/\nu$, dimensionless coordinate; $\delta^+ = V^*\delta/\nu$, dimensionless thickness of the effective viscous layer.

REFERENCES

1. V. Ye. Nakoryakov, A. P. Burdukov, O. N. Kashinskii, and P. I. Geshev, Electrodiffusion Method of Investigating the Local Structure of Turbulent Flows [in Russian], Novosibirsk (1986).
2. T. J. Hanratty, L. G. Chorn, and D. T. Hatziavramidis, Phys. Fluids, **20**, No. 10, Part 11, 112-119 (1977).
3. V. Ye. Nakoryakov, O. N. Kashinskii, B. K. Koz'menko, and R. S. Gorelik, Izv. Sib. Otd. Akad. Nauk SSSR, Ser. Tekhn. Nauk, No. 16, Issue 3, 15-20 (1986).
4. V. V. Randin, Fifth All-Union School on "Modern Problems of Thermal Physics," Abstracts of Papers [in Russian], 126-127 (1988).
5. K. Blake, J. Fluid Mech., **44**, 637 (1970).
6. W. R. B. Morrison, K. J. Bullock, and R. E. Knonauer, J. Fluid Mech., **47**, Part 4, 639-656 (1971).
7. H. P. Bakewell and J. L. Lumley, Phys. Fluids, **10**, No. 9, 1180-1189 (1967).
8. J. Steruberg, J. Fluid Mech., **13**, 241-271 (1962).
9. P. S. Klebanoff, Characteristics of Turbulence in a Boundary Layer with Zero Pressure Gradients, NACA Tech. Note No. 3178 (1954).
10. A. V. Chinak, in: Gas-Liquid Flows: Collected Scientific Papers (V. G. Gasenko and M. S. Iskakov (eds.)) [in Russian], Novosibirsk (1990), pp. 102-108.